K–12 Mathematics Education: How Much Common Ground Is There?

By Anthony Ralston

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In the August/September 2005 issue of FOCUS there was a brief summary [1] of a document entitled "Finding Common Ground in K-12 Mathematcs Education" (hereafter CG), whose full text may be found at http://www.maa.org/common-ground [2]. The authors of CG are two research mathematicians, three mathematics educators and the convener (-1. (of meeting) convocante mf -2. Br (in trade union) representante mf sindical) of the group, who is a senior vice-president and math and science policy advisor for a major American technology corporation and who has a Ph. D. in applied mathematics.

There has been much controversy about American school mathematics in, at least, the past 15 years. The players have been, roughly speaking, the research mathematics community on one side, and the mathematics education community on the other. Thus, trying to find common ground between these two communities would appear to be a valuable exercise. And, indeed, much that is in CG will seem unexceptionable to almost all readers of FOCUS. But creating a document of consensus among six individuals who represent various points of view on the matter being discussed is fraught (tenso(a), tirante) with difficulties.

One difficulty is that the attempt to be unexceptionable too easily results in blandness. ()sosería f, insulsez f -2. (of person) insulsez). Another is that, although the authors have certainly wished to avoid ambiguity, they have not always succeeded. A final difficulty is that when, occasionally, there is a definite recommendation, a group of six — *any* six — is just not enough to assure that there will not be significant disagreement in the communities they are addressing. The authors of CG have not avoided these pitfalls.

There is no need to say much about the blandness of some of the statements in CG since it is inevitable that there will be some in a document like this. But statements like "All students must have a solid grounding in mathematics to function effectively in today's world", "Students must be able to formulate and solve problems", and "Teaching mathematics effectively depends on a solid understanding of the material" would perhaps better have been omitted or, preceded by "Since", they could have been in each case attached to the sentence that follows.

I am sure the authors of CG strove mightily to avoid ambiguity. Here are two examples when I think they have not succeeded (at least for me).

(i) "Certain procedures and algorithms in mathematics are so basic and have such wide application that they should be practiced to the point of automaticity." But, without examples, what can this mean? The only example given is this one: "Computational fluency in whole number arithmetic is vital." What other procedures and algorithms, if any, should be automatic? And what about "computational fluency in whole number arithmetic"? Does this mean, for example, that students should be expected to be fluent with the traditional algorithm for long division? At most a small fraction of students have ever become "fluent" with this algorithm. And with calculators so easily available, what expectation can there be that more than a small fraction of students will become fluent in the future? And why just "whole number arithmetic"? Is arithmetic with decimal numbers less important than whole number arithmetic? Certainly not in the workplace.

(ii) "Calculators can have a useful role even in the lower grades, but they must be used carefully, so as not to impede the acquisition of fluency of basic facts and computational procedures." Since some of the authors have in the past opposed any use of calculators in K–6, this is a step forward. But what is the second portion ("but …") supposed to imply? If only that calculators should not be used mindlessly or for one-digit arithmetic, then this is a triviality not worth saying. If something more than this, then what? One suspects that, in order to accede to the first portion of this sentence, some of the authors insisted on the ambiguous second portion. This is a standard problem with consensus documents.

Some of the above might be viewed as mere quibbling (peros o sofismas de distración) although I think it is more than this. In any case, the examples below are of issues that will surely elicit (sacar, obtener, generar) disagreement with CG among a substantial number of readers of FOCUS.

(i) "By the time they leave high school, a majority of students should have studied calculus." Leave aside the fact that this— or anything close to it — cannot be achieved in any foreseeable future. Leave aside also the fact that many students who now study calculus in high school come away from it with little understanding and little more than an ability to perform mechanically various algorithms, all of which can be done better on a calculator. But, anyhow, why would you wish half the students to have studied calculus? Too much of the mathematics community has failed to come to terms with the fact that discrete mathematics is (almost?) as good an entrée to college mathematics as calculus. Not to recognize this in a document such as this is to arouse the suspicion that too many of the authors are living in the past. If they had said "…a majority of students should have studied first year college mathematics", that would at least have been a defensible aspiration. I would still not have agreed with it on the grounds of unattainability (inalcanzabilidad) but, at least, the document would have sounded like it had had input from some younger mathematicians.

(ii) "Students should be able to use the basic algorithms of whole number arithmetic fluently, and they should understand how and why the algorithms work." This statement, no doubt, is to forestall (anticiparse a, adelantarse a   -2. (prevent) impedir) people like me who have advocated abandoning traditional instruction in paper-and-pencil arithmetic [3]. But it sounds like voices from another century (the 20th!) to expect that most students will become fluent in the traditional algorithms when it is obvious that, outside of school, many (almost all?) students will use calculators to do their arithmetic homework, no matter how much their teachers inveigh (lanzar invectivas contra) against it. And is there any chance that a significant number of students will "understand how and why the algorithms work"? <>

(iii) "The arithmetic of fractions is important as a foundation for algebra." Many of you may think this statement is innocuous but I don't. No one doubts that any non-trivial study of algebra must involve arithmetic with algebraic fractions. But while students should learn about reciprocals and the conversion of fractions to decimals and vice versa before college, it does not follow that prior study of the *arithmetic* of numerical fractions, even if still remembered by the time algebra is studied, is a good or necessary prelude to this. Indeed, the addition, subtraction and, particularly, the division of algebraic fractions [4] is rather easier than the same operations for numerical fractions. So what if students come to algebra without knowing the arithmetic of numerical fractions? Just teach it as part of the algebra course. Not only are the algorithms generally easier but the more mature high school students will learn them more rapidly than middle school students. Then, if you wish, apply the algebraic algorithms to numbers.<>It is, I believe, almost surely futile at this time to attempt to find significant agreement between the research mathematics and mathematics education communities on the major issues confronting American school mathematics education. The disagreements on various matters — curriculum and technology being perhaps the most profound and obvious — are just too deep at this time to allow any non-trivial consensus. <>

Before such consensus can be reasonably attempted there will have to be, at least, a level of respect in both communities for the other that will mean that inevitable disagreements need not erupt into shouting matches ()the debate ended up as a shouting match (el debate acabó a grito pelado) . The CG document evinces (evidenciar) such respect but it is far from universal among research mathematicians or mathematics educators. Mathematics educators must accept that professional mathematicians, research and otherwise, through their experience and insights, have the potential to offer much to school mathematics education. Research mathematicians need to understand that college and university mathematics educators generally, as well as many secondary school mathematics teachers, know and understand school mathematics. And research mathematicians will have to accept that the mathematics education community generally knows considerably more than they do about appropriate pedagogy for school mathematics.

References

1. Pearson, M., *Finding Common Ground in K-12 Mathematics, FOCUS*, Vol. 25, No. 6, 2005, p. 40.
2. Ball, D. L., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R. J., Schmid, W., Schaar, R, *Reaching for Common Ground in K-12 Mathematics Education*, http://www.maa.org/common-ground; also in *Notices of the AMS,* Vol. 52, No. 9, October 2005, pp. 1055-1058.
3. Ralston, A., *Let's Abolish Pencil-and-Paper Arithmetic*, http://www.doc.ac.ic.uk/~ar9/abolpub.htm.
4. Ralston, A., *The Case Against Long Division*, http://www.doc.ic.ac.uk/~ar9/LDApaper2.html.

Response

As the convener of the team of research mathematics and mathematics educators who are the authors of *Finding Common Ground in K-12 Mathematics Education*, (FCG) I felt it was necessary to comment on the above-mentioned piece. Speaking for the other authors, I must thank Anthony Ralston for his in-depth analysis of our document. We will certainly consider his remarks as we continue and expand our work. They are very helpful.

However, I must comment on the last two paragraphs in his piece. As for finding "significant agreement," FCG is an existence proof that such agreement can be developed from mutual understanding starting with a good diversity of expert opinions. When I convened the group, many confided that they thought the group would agree on very little; after defining terms and working on the issues, however, the group agreed on almost everything. Will research mathematicians and mathematics educators agree on everything? No, they will not. Not all research mathematicians (or mathematics educators) agree on everything, but it is the dialogue and development of what they can agree upon that is the key. It is my belief that there is enough significant agreement that as a group, we can move forward in educating our youth in mathematics, which is a crisis area for the United States and is one that cannot wait to be solved.

Finally, from what I have seen, there is a lot more respect between both communities than I was led to believe when I started this work. I have witnessed a great deal of cooperation and understanding to solve the common problems of K-12 mathematics education. It is interesting that on the same page as the two paragraphs, there is an advertisement for the Institute of Advanced Studies' Park City Mathematics Institute. If you look at the participants and organizers, you get a glimpse of the broad spectrum of participation around the education theme of "Knowledge for Teaching Mathematics." Thus the small group I led does not represent the only ongoing discussion aimed at bringing the community together to find areas of agreement and to approach disagreement amicably and respectfully. Such conversations are ongoing and expanding. The community must continue to move beyond questions of respect to get the job done.

-Richard Schaar, January 2006