MARTIN GARDNER, ALICE AND THE GRAVITY LAW

(English short version)

Diego Pareja-Heredia
Department of Mathematics, Universidad del Quindío. Armenia. Colombia

1. Introduction.

This communication is a short account of a teaching project that Professor Jairo Urrea Henao, and the speaker, have being testing inside Universidad Del Quindío, where we are lecturing in philosophy and mathematics, respectively. The central aim of our work is to present to students and teachers of science education, some mathematical and physical topics in a context, bordering literature, philosophy and history.

The methodology we are following has, as a backbone, in one hand, an article from a mathematical journal[1], and in the other side a classical literary work, which creates the humanistic balance to motivate and introduce the physical or mathematical concept, we want to present into the classroom.

For the specific case of *Gravity Law*, the motivation came from *Alice in Wonderland* [1] by Lewis Carroll, together with Martin Gardner's book, *The Annotated Alice* [2]. The main article for our purpose became *The Gravity of Hades* [4] by Andrew J. Simoson, published in the *Mathematics Magazine* in 2002.

The two *Alices*[2] by Carroll are truly, gold mines, where we can get invaluable motivational material relating with science and mathematics. The starting point for our work were the first pages of *Alice's Adventures in Wonderland*, more exactly, the passage, when Alice runs after a rabbit through a hole and suddenly the hole went down vertically into the earth, and the girl, while falling, ask herself questions about the way she is falling toward the Earth's center, and how long it takes to reach it.

Prior the presentation of the main theme let me introduce the *dramatis personae* of our play story, namely: Martin Gardner, Lewis Carroll and Alice.

2. Martin Gardner.

Martin Gardner (1914 -) is a very know writer with about sixty published books in a variety of fields, ranging from science fiction and philosophy to mathematics and literature. Beginning at the fifties of the past century, his column *Mathematical Games* published at the journal ***Scientific American***, was very influential in the mathematical community. From those columns, many books sparkled out, which fortunately, are in print today and they keep their freshness and motivating style as the original columns.

Martin Gardner is a Lewis Carroll's admirer and he has written several books analyzing the work and life of the author of *Alice's Adventures in Wonderland.* It was through his book *The Annotated Alice* that we arrived to the problem of considering changes in the behavior of gravity, according to models of internal Earth structure. The *Annotated Alice* (Fig. 1), a best seller with

[1] We refer to *The American Mathematical Monthly*, *The Mathematics Magazine, The College Mathematical Journal, The Notices* and *The Bulletin of the American Mathematical Society,* among others.
[2] We mean: *Alice's Adventures in Wonderland* and *Through the Looking-Glass and What Alice Found There.*

more than a million of sold copies, contains *Alice's Adventures in Wonderland* and *Through the Looking-Glass,* with illuminating notes written by Gardner, which makes the fantasy books of Carroll a marvelous source of motivational aids for the classroom.

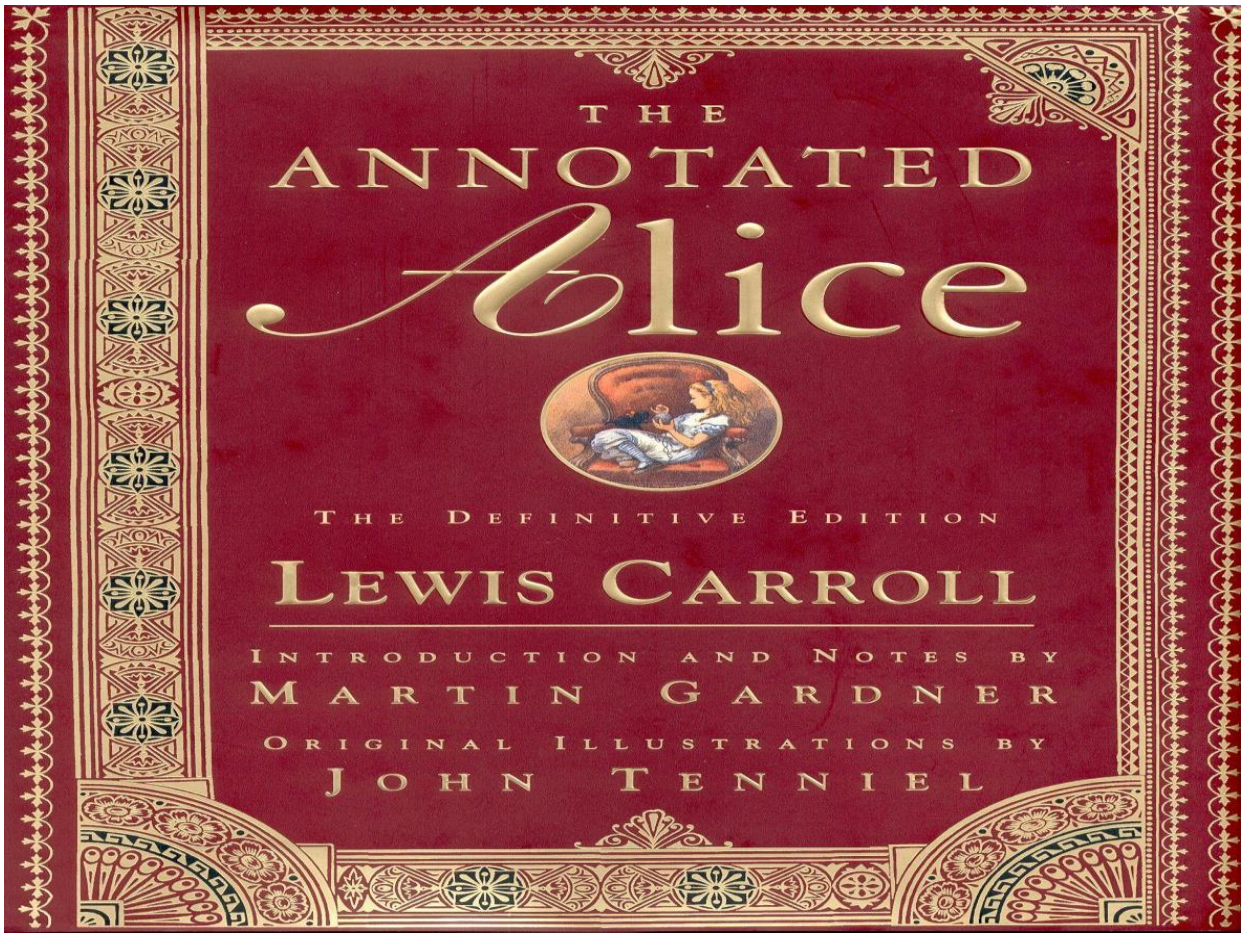

Fig. 1

Gardner had a philosophical formation during his undergraduate years at the University of Chicago (Class 1936). After returning from four years in the Navy, he attended the Seminar of Science under the leadership of Rudolf Carnap. The magneto phonic recordings of this seminar were edited and published by Gardner in the form of the book *Philosophical Foundations of Physics* and now appears under the title *Introduction to the Philosophy of Science*. Some titles of his intellectual production are related with an acid critic of pseudo science, particularly in fields like paranormal powers, future prediction and topics of politics and religion. Among his most known books about pseudo science are *Fads and Fallacies in the Name of Science* (1952), *Science: Good, Bad and Bogus* (1981), *The New Age: Notes of a Fringe-Watcher* (1988) and *Weird Water and Fuzzy Logic: More Notes of a Fringe-Watcher* (1996).

In an essay in the New York Review of Books, Stephen Jay Gould said about Martin Gardner "(he) has become a priceless national resource," a writer "who can combine wit, penetrating analysis, sharp prose, and sweet reason into an expansive view that expunges nonsense without stifling innovation, and that presents the excitement and humanity of science in a positive way".

Trying to create this "excitement and humanity of science" is at the focus of our present work.

3. Alice and Lewis Carroll.
Charles Lutwidge Dodgson (1832-1898) was the baptism name of Lewis Carroll. He had a wide background in mathematics and logic. Actually, his books in those themes were known in his time. *The Game of Logic; Euclid and his Modern Rivals; A Logic Paradox* and *What the Turtle said to Aquiles*, are a few titles among his mathematical production.

Carroll loved literature, and besides the two *Alices*, he wrote, *Sylvie and Bruno* and *To the Haunting of the Snark*. He was also photography's pioneer and maintained a wide collection of photographs of important personalities of his time.

Even thought the two *Alices* were conceived as books for children; its reading, is also valuable for educators, since in them, appears many illuminating ideas to use in teaching as motivational material, to introduce concepts and topics in science, literature and philosophy. It's very interesting to note that *nonsense* introduced by Carroll, is like a simulation of a reality we live under the influence of dreaming, where the logic of that "reality" differs from logic of awakening. Inside this fictitious reality, occurs strange things so that, when contrasted them, with physical reality, gives us the possibility to see this latter one, more clearly and more real.

Lewis Carroll's life and work have been studied extensively, at such a point, that Martin Gardner has in his *Annotated Alice* more than one hundred items related with those topics. Alice, the main feature of the books of Carroll, has been considered as the *leit-motif* for the behavior of youngsters, with all of their psychological complexity, their fantasies and their fears.

Several Lewis Carroll Societies have been created around the world to promote and keep alive the ideas and the literary and scientific legacy of Alice's creator.

4. Alice and the main question.
At the beginning of *Alice's Adventures in Wonderland*, Alice "…run across the field after it (the rabbit), and was in time to see it pop down a large rabbit-hole under the hedge…" and then "… The rabbit-hole went straight on, like a tunnel for some way, and then dipped suddenly down, so suddenly that Alice had not a moment to think about stopping herself before she found falling down what seemed to be a very deep well…."

When Alice was falling, she asks herself, "I wonder how many miles I've fallen by this time? "She said aloud". "I must be getting somewhere near the centre of the earth. Let me see: that would be four thousand miles down, I think". From these Alice's thoughts we may state up the main question:

What is the behavior of gravity inside the Earth?

To answer this question we may imagine the rabbit-hole all through the earth and assume that it goes from pole to pole and upon ideal conditions, without air friction and without the influence of the earth rotation. From the mathematical and physical point of view, we start with Newton's classical mechanics, namely with the formulas:

$$F = GmM/x^2 \quad \text{and} \quad F=ma$$

Here G is the gravity constant, m and M the mass of two bodies been attracted through gravitational force and x the distance between the bodies. In the right side formula, F means force m, mass and a stands for acceleration. We begin with the figure

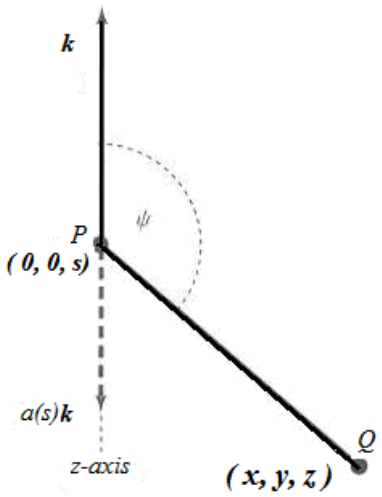

***Fig.* 2**

We see here: the unitary vector $\boldsymbol{k}$ = (0, 0, 1), through the imaginary hole, where the movement takes place. P= (0, 0, s) represents the position of the point where the falling object is at an instant t. Q= (x, y, z) is an arbitrary point of the earth sphere, Ψ is the angle between $\boldsymbol{k}$ and the vector PQ and $a(s)\boldsymbol{k}$ represents the component of gravitational force in the $\boldsymbol{k}$ direction.

Applying the rules of elementary vector calculus and some elementary physics, we get

$$a(s) = \frac{Gm(z-s)}{(x^2+y^2+(z-s)^2)^{\frac{3}{2}}}. \quad \textbf{(1)}$$

The gravitational acceleration $a(s)$, at a distance s of the center of the sphere, is referred to the point P and in the direction of vector $\boldsymbol{k}$**.** For ideal planets, formed by spherical concentric shells **S** of mass M, with constant density and radius r, $a(s)$ is calculated according to the following expression.

$$a(s) = \begin{cases} 0, & \text{If } 0 \le s < r, \\ -\dfrac{GM}{2s^2}, & \text{If } s = r, \\ -\dfrac{GM}{s^2}, & \text{If } s > r. \end{cases} \quad \textbf{(2)}$$

Derivation of these formulas can be found actually in classical mechanics books. We follow here the Simoson paper just mentioned, to find, after some calculus operations the expression:

$$a(s) = \frac{GM}{2s^2}\left(\frac{r-s}{\sqrt{(r-s)^2}} - \frac{r+s}{\sqrt{(r+s)^2}}\right) \quad \textbf{(3)}$$

Which is the same as (2) whenever $0 \leq s < r$, and $s > r$**.** For the case $r = s$, we have to use an easy trick before we get (3).

5. Models for the Earth interior.

It is not known exactly the structure of the inner part of the earth. However we can make some assumptions to deduce some consequences for acceleration due to gravity and consequently to get formulas for velocity and time when the falling body reach earth center. For several model samples of the Earth interior see the Simoson paper [4]. We present here a few cases.

A GENERAL MODEL WITH DENSITY

Let **S** be a sphere with radius R, mass M, center **0** and density δ**(**s**),** at s unities from the origin **0**. This means that, **S** is spherically symmetric related to density. Then

$$a(s) = \begin{cases} -\dfrac{4\pi G}{s^2}\displaystyle\int_0^s \rho^2\delta(\rho)\,d\rho, & \text{Si } 0 \leq s \leq R, \\ -\dfrac{GM}{s^2}, & \text{Si } s \geq R. \end{cases} \quad \textbf{(4)}$$

The mass for the shell **S** of radius ρ is $4\pi\rho^2\delta(\rho)$ (Think of **S** as a spherical layer of infinitesimal thickness, whose surface is $4\pi\rho^2$ and its spherical density is $\delta(\rho)$**).** So using **(2)** we get

$$a(s) = \begin{cases} -\displaystyle\int_0^s \frac{4G\pi\rho^2\delta(\rho)}{s^2}\,d\rho, & \text{Si } 0 \leq s \leq R, \\ -\displaystyle\int_0^R \frac{4G\pi\rho^2\delta(\rho)}{s^2}\,d\rho & \text{Si } s \geq R, \end{cases}$$

This is the same as (4), since M, the mass, is included in the above integral.

INTUITIVE AND MITICAL MODELS FOR ACCELERATION**.**

Intuition suggests that gravity remains constant as we descent toward earth center. According to Ovid(I Cent. BC) in *Metamorphosis,* mythical heroes as Orpheus claiming his wife from Hades and Hercules capturing Cerberus, does share our intuition that gravity does not change when we descent inside the earth.

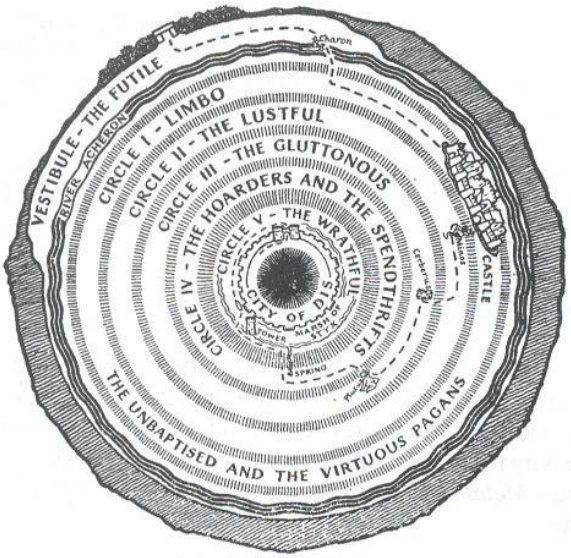

***Fig.* 3**

Dante Allighieri in *The Divine Comedy* gives us the impression of how hard is the ascending or descending through the Hell circles (*Fig. 3*) with his ghost guide Virgil. Dante mentions that Satan (the devil) is exactly at earth center and Plutarch (I century AD) suggests that if a man stays at the earth center, his head and his foot are simultaneously above.

Accepting that gravity is constant implies that $\delta(s)$, must satisfy, according to (4), the following relationship:

$$-\frac{4\pi G}{s^2}\int_0^s \rho^2\delta(\rho)\,d\rho = C,$$

Here C is a constant. Multiplying both sides by s^2, and deriving implicitly, we get,

$$\delta(s)=k/s,\ k \text{ a constant.}$$

This short formula relates density δ with s, the distance to the Earth center. That shows that this ancient earth model of constant gravity implies that density and distance at earth center are in inverse proportion.

A MODEL WITH CONSTANT DENSITY

This model results when we assume a constant density, that is, when mass is uniformly distributed inside the earth. If the density is δ and M is the mass of the sphere $\boldsymbol{S}$ of radius R, we find using (4)

$$a(s) = \begin{cases} -\dfrac{4\pi G\delta s}{3}, & \text{Si } 0 \le s \le R, \\ -\dfrac{GM}{s^2} & \text{Si } s \ge R. \end{cases} \quad \textbf{(5)}$$

This formula shows that gravity and the distance to the earth's center are proportional up to reaching the surface, and from here up, gravity will be proportional inverse to the square of the distance *s* to the earth's center.

THE HALLEY MODEL

Edmond Halley (1656-1742), known as the geophysics´ father, suggested that Earth could be 4/9 hollow with an outer shell 500 miles thick. Inside of it, would have shells with radii similar to those of Venus, Mars and Mercury with spaces between them, even containing atmospheres with light produced by magnetic fields like the aurora borealis flashes. Furthermore he adventured to suggest the possibility of life over the surface of these inner shells. A Halley's portrait made in 1736 shows him with holding a hollow earth model (Fig. 3).

Halley's model was taken as an inspiration by Edgar Rice Burroughs (the creator of T*arzan*) for writing his novel *At the Earth's Cor*e, where he describes a country called *Pellucidar*, down the earth, at 500 miles deep.

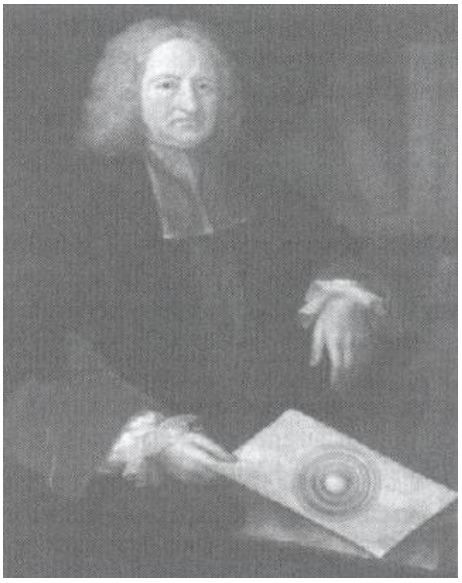

***Fig.* 4**

Following Halley's model we can calculate gravity inside one of the shells, say ***S***, between radii *r* and *R.* If we assume uniform density δ and mass *M,* formula (3) give us

$$a(s) = \begin{cases} 0, & \text{Si } 0 \le s \le r \\ -\frac{4}{3}\pi G\delta\left(s - \frac{r^3}{s^2}\right), & \text{Si } r \le s \le R, \\ -\frac{GM}{s^2} & \text{Si } s \ge R. \end{cases} \quad \textbf{(6)}$$

It follows from this expression that gravity is zero inside the shell ***S***.

HOOKE´S ONION MODEL.

Robert Hooke (1635-1703), considered Earth as formed of two homogeneous parts, a core with mass M_1 density δ_1 and a mantle with density δ_2 and mass M_2. For this case we find for acceleration the following formulas:

$$a(s) = \begin{cases} -\dfrac{4\pi G\delta_1 s}{3}, & \text{Si } 0 \le s \le r, \\ -\dfrac{GM_1}{s^2} - \dfrac{4}{3}\pi G\delta_2\left(s - \dfrac{r^3}{s^2}\right), & \text{Si } r \le s \le R \\ -\dfrac{G(M_1 + M_2)}{s^2}, & \text{Si } s \ge R. \end{cases}$$

6. Answer to Alice's questions.

Using the precedent models we now can calculate velocity and time at earth center for a falling body inside the earth.

With some elementary calculus we get

$$v(s) = -\sqrt{2\int_R^s a\,dr}. \qquad T = \int_0^R \frac{-1}{v}\,ds.$$

Since we know $a(s)$ for the models described above, we can evaluate velocity and time when the falling body reach earth center. For the cases of constant density and constant gravity we get:

For *Constant density:* $v(0)$ = -7.9 Km./seg. T= 21.2 Min.

For *Constant Gravity*: $v(0)$ = – 11.2 Km./seg. T = 19 Min.

For a *Seismic Model of the Earth*, Simoson [4] gives,
$v(0)$ = –9.8 Km./seg. T = 19.2 Min.

The complete Spanish version of this communication can be read at [3].

REFERENCES

1 - CARROLL, Lewis. *Aventuras de Alicia en el País de las Maravillas,* Alianza Editorial. Madrid. 1974. Trd. Jaime de Ojeda**.**

2 - GARDNER, Martin. *The Annotated Alice*. *The Definitive Edition*. W. W. Norton & Company. New York. 2000.

3 - PAREJA-HEREDIA, Diego. et al. *Martin Gardner Alicia y la Ley de la Gravedad.* www.matematicasyfilosofiaenelaula.info.

4 - SIMOSON, Andrew, J. *The Gravity of Hades*. Mathematics Magazine. Vol. 75 No. 5, December 2002.