A NEW DEFINITION OF NATURAL NUMBERS
And its Incidence on Teaching Mathematics

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"To *go wrong in one's own way is better than to go right in someone else's*."
— Fyodor Dostoyevsky, **Crime and Punishment**

Introduction

The main purpose of this paper is to introduce at an elementary school level a formal definition of the concept of natural number. To do this, we shall use very intuitive ideas such as copying and pasting. The starting point is a set of ten basic symbols and the analogy of addition and multiplication with pasting and copying. Most of the material here is elementary in its spirit, and it is thought for arithmetic teachers interested in seeing mathematics in a quite different way from that presented to us in the traditional curriculum.

The philosophy that animates this article has its origin in Gottlog Frege[1], who was the first mathematician to understand the importance of the concept of definition in mathematics. As a matter of fact, one time he said: "is above all number, which must be defined or recognized as indefinable." His interest in defining number is related with proving the arithmetical laws. He introduces the concept of ancestry in order to define the concept of natural number[2].

Why a new definition could be useful? In our case, at least because, this new definition for natural numbers let us give another view of prime numbers without consider the traditional way of using division for this task. We use factoring instead. We can also understand arithmetic algorithms in a reasonable way and do not through memorizing boring routines.

Our elementary approach combines arithmetic and algebra; from basic operations to solutions of equations, including equations of degree greater than two, when some of their solutions are in **Q**, the set of rational numbers. Using our methodology, it is possible to introduce early in teaching mathematics, notions from linear algebra and advanced mathematics. This approach also let us to introduce at elementary school, numbers sets like, **Z**, **Q** and **R,** including the set of algebraic numbers and concepts, say, as irrationality, countability and uncountability.

This is not a technical paper with a long list of definitions, lemmas and theorems. Instead I have chosen the way of introducing all material through examples and motivations to go further in studying more deep ideas. The starting point is the creation of some mathematical objects called polynomials, namely the sequence of products and sums of elements in a fix set of ten symbols, called digits and an additional symbol for the cardinal of this set.

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[1] See the interesting paper: WAGNER, S. *Frege's Definition of Number*
Notre Dame Journal of Formal Logic. Volume 24, Number 1, January 1983. Available at the web

[2]See my epistemology notes in:
http://matematicasyfilosofiaenelaula.info/Epistemologia%202009/Los%20Números%20Naturales%20y%20el%20Concepto%20de%20Buena%20Ordenacion.pdf

Equivalence Classes

Dividing math objects in classes is a common methodology. For instance, natural numbers can be seen as the union of two disjoint sets: odd and even numbers. Even numbers are multiples of 2, while odd numbers are not.

We are interested here, in some special classes $\mathscr{F}$, namely, classes with the property that, whenever we have elements *a* ,*b, c, d,* …, in the class and a relation *R* defined in the class, it must satisfy:

Reflexivity: *aRa*
Symmetry: *aRb,* implies, *bRa.*
Transitivity: If *aRb*, and, *bRc*, then, *aRc.*

If *R* satisfies the above properties in a given set, this set is partitioned by *R* in disjoint sets, in such way that their union reproduces the initial set. When two members *a* and *b* are in $\mathscr{F}$, we say that *a* and *b* are **congruent** or **equivalent** module R.

The set ℕ of natural numbers {0, 1, 2,…} has being defined in many ways through the last millennia. Here we propose a new constructive definition trying to arrive to a different way to see natural numbers and their properties. To begin with, let us use an intuitive approach, now very used in the computer age: copying and pasting. Copying in our context means repeating a process or action; in the numerical case, this means, multiplication. Similarly, pasting means put together a pair of objects and in the numeric case, it means addition.

Pasting *a, b* in ℕ will mean $a + b$ and copying *a, b* in ℕ, should mean, take *a* copies of *b,* or *b* copies of *a,* in symbols, $a \cdot b$ or simply, *ab*. For instance, we can mean, paste 2 and 3, as, 2 + 3 or 3 + 2; and copy 2 and 3, as, 2 · 3, or 3 · 2. Copying *a, a*; *a, a, a*; …, will be expressed as: a^2, a^3, …, respectively.

Copying and pasting objects in a set are reversing actions, namely, *a* pasting *b* is same that *b* pasting *a*; this is also true for copying. Consequently the commutative law is inherited by addition and multiplication of natural numbers. Similar argument validates associative and distributive laws in the set ℕ of natural numbers. Symbolically, for *a*, *b* and *c* in ℕ:

Commutative Law. $a + b = b + a$; and, $a \cdot b = b \cdot a$

Associative Law. $(a + b) + c = a + (b + c)$; and, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Distributive Law. $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$

Let's take for granted the existence of a basic set of symbols to represent graphically the set ℕ, of natural numbers, say: D = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}[3]. We also assume the existence of an extra symbol, say, *x*, to design the number of elements of D. We suppose that it is possible to define two inner operations: +, and, · in ℕ, in such way that if $a, b \in \mathbb{N}$ then, $a + b \in \mathbb{N}$ and, $a \cdot b \in \mathbb{N}$.

[3] We can also select different bases as short as {0, 1} to represent all natural numbers.

Representing ℕ as a family of some special polynomials

The set D, defined above, is called a base for representing natural numbers. This set has ten elements and the representation of natural numbers using this base is called a decimal representation of ℕ. The origin of this representation goes back to ancient India, and were the Arabs around Middle Ages who brought this decimal system to west. Let us call x, the cardinal of D, namely, the number of elements of D. In our case, $x = 10$. The powers of ten, 10^0, 10^1, 10^2, 10^3, ... , will be represented by 1, x, x^2, x^3, …. Note that for the case of our decimal positional system,

$$x^n = 10x^{n-1}$$

This means that, the relative n-th position (counted from right to left) in the numeral has the same value of 10 times the (n–1) position. This syntactic property will let us represent natural numbers in several forms using polynomials.

The sequence of natural numbers (counting numbers) using copying and pasting appears as: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, x, $x + 1$, …, $x + 9$, $2x$, $2x + 1$, …, $2x + 9$, $3x$, $3x + 1$, …, $9x + 9$, x^2, $x^2 + 1$, …, $2x^2$, $2x^2 + 1$,…, $9x^2 + 9$, …, $9x^2 + 9x + 9$, x^3, $x^3 + 1$, … .

Number 2786, using copying and pasting looks like: $2x^3 + 7x^2 + 8x + 6$. In this example, we have 6 units, 8 tens, 7 hundreds, and 2 thousands. Forgetting that $x = 10$, an expression as, $2x^3 + 7x^2 + x + 6$, looks like an algebraic object called one variable polynomial. Here the indeterminate x takes the value of ten; in a general context, "x" could be another kind of number, a function or something else. In other papers[4] I have proposed the way to perform arithmetic operations, using numbers expressed as polynomials.

Definition of Standard Polynomials

Let ℕ[x] be the set of all one variable polynomials with coefficients in ℕ. Let a, be a natural number written in its standard decimal representation, say, $a = (a_n a_{n-1} \ldots a_1 a_0)$, we define inductively a as:

$a = (a_n a_{n-1} \ldots a_1 a_0) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 = \sum_{i=0}^{i=n} a_{n-i}x^{n-i}$, with $x = 10$ and $a_i \in \{0, 1, 2, ..., 9\}$.

We call polynomial $P_a(x) = \sum_{i=0}^{i=n} a_{n-i}x^{n-i}$, $a_i \in \{0, 1, 2, ..., 9\}$, the *standard polynomial* associated to a.

We will use $\mathscr{S}$ to represent the family of all sets of standard polynomials. Clearly $\mathscr{S}$ is inside ℕ[x].

[4] See for example:
http://matematicasyfilosofiaenelaula.info/articulos/Beginning_Abstract_Algebra_at_Elementary_School.pdf and http://matematicasyfilosofiaenelaula.info/articulos/ROOTS%20AND%20STEMS%20V.1.pdf

This means that a natural number is uniquely associated to a standard polynomial and vice versa, any standard polynomial is uniquely associated to a natural number. For instance, the standard polynomial associated to 23578 is $2x^4 + 3x^3 + 5x^2 + 7x + 8$. Also a polynomial like $P(x) = 2x^4 + 3x^3 + 5x^2 + 7x + 8$, is associated to the natural number 23578.

$\mathbb{N}[x]$ is much more than the class of standard polynomials, since polynomials like $12x^3 + 28x^2 + 2x + 43$ belongs to $\mathbb{N}[x]$, but it is not a standard polynomial.

For each natural number *a* we can also associate to it, all polynomials P(x) in $\mathbb{N}[x]$, such that P(10) = *a*. For instance, the number $a = 124$ is associated to its standard polynomial $P(x) = x^2 + 2x + 4$. However we can also associate to it, polynomials like, $x^2 + 24$, $12x + 4$, even the constant polynomial $P(x) = 124$, because, when we replace x for 10 we get 124.

In a general context, to a number *a*, we can associate all a family of polynomials of a variable x. More exactly, all polynomials P, such that, P(10) = *a*. These classes $\mathscr{F}$, are equivalent classes, that is, if *f* and *g*, belong to $\mathscr{F}$, then $f(10) = g(10) = a$. The equivalence relation R is defined here as: P is equivalent to Q if and only if P(10) = Q(10) = *a*. We say in this case that *f* and *g* are **congruent module** *a*. When we previously know that *f* and *g* are in $\mathscr{F}$, we will say that $f = g$.

Definition of Congruent Polynomials

We say that two polynomials P and Q are congruent module *a* (or equivalent module *a*), whenever P(10) = Q(10) = *a*.

A pair of polynomials may be congruent but not necessarily equal. It is the case when, $P(x) = x^2 + 2x + 4$ and $Q(x) = 12x + 4$. P and Q are congruent module 124, but they are not equal in $\mathbb{N}[x]$; the first one is a quadratic polynomial; the second one is a linear polynomial indeed, one is different from the other. When we are inside an equivalent class we consider the elements on it, as equivalent; and abusing of language, we can say that they are equals. I hope this ambiguity does not create confusion.

Definition of natural numbers using equivalent classes

We define number *a*, written ($a_n a_{n-1} \ldots a_1 a_0$), as the family $\mathscr{F}$ of all equivalent polynomials module *a*, symbolically:

$\Omega_a = \{ \sum_{j=0}^{j=n} b_{n-j} x^{n-j} \in \mathbb{N}[x]$, except, $P(x) = a$, such that, any pair P, Q of polynomials in the family $\mathscr{F}$ satisfies: $P(10) = Q(10) = a\}$.

The relation R in the congruence definition mentioned above corresponds to the fact that P(10) = *a*, for all polynomials defined in the family.

According to the above definition we can identify the number *a*, with Ω_a, the set of all polynomials congruent to *a*.

The preceding definition can be extended to include coefficients in **Z**, when natural numbers and polynomials be defined. Integers are defined as the set of all solutions of linear equations like $ax + b = 0$, whenever, $a, b \in \mathbb{N}$, $a \neq 0$.

For instance, number 326 could be defined as:

$326 = \Omega_{326} = \{3x^2 + 2x + 6, 3x^2 + 26, 32x + 6, 31x + 16, \ldots, 3x^2 + 3x - 4, \ldots\}$.

All the above polynomials have a numerical value of 326, whenever, $x = 10$, namely, all of they are congruent or equivalent, module 326.

When we are inside Ω_{124}, $P(x) = x^2 + 2x + 4$ and $Q(x) = 12x + 4$ are congruent module 124, or equivalent module 124, since $P(10) = Q(10) = 124$. Inside the class Ω_{124}, we usually say: $x^2 + 2x + 4 \equiv 12x + 4$(module 124) and when there is not risk of confusion, we will say: $x^2 + 2x + 4 = 12x + 4$.

The above definition let us to introduce prime numbers without making any allusion to division. The reason is that through polynomial usage, factoring is the inverse process of multiplication. This means that, multiplication of polynomials in Ω_a may be reversed using factoring.

Suppose, $a = 2263$. Assuming we are in Ω_{2263}, taking $P(x) = 3x + 1$ and $Q(x) = 7x + 3$, the product, $P(x)Q(x) = (3x + 1)(7x + 3) = 21x^2 + 7x + 9x + 3 = 21x^2 + 16x + 3 = 22x^2 + 6x + 3 = 2x^3 + 2x^2 + 6x + 3 = 2263$. This last product can be factored reversing the process to recover $P(x)$ and $Q(x)$ and so $2263 = P(10)Q(10) = 31 \cdot 73$. From here we conclude that $2263 = 31 \cdot 73$ and so 2263 is a composite number [5].

If a number a is associated to a standard polynomial $S(x) = P(x)Q(x)$ in Ω_a, then $a = P(10)Q(10)$ is a composite number. From here we can get the following definition.

Definition of Composite and Prime Numbers

A number a, defined using Ω_a, is said to be *composite* if there exists a polynomial P in Ω_a, which can be factored. Otherwise a will be called *prime*.

Example

The number $747 = \Omega_{747} = \{7x^2 + 4x + 7, 6x^2 + 14x + 7, 6x^2 + 12x + 27, \cdots\}$. Since the polynomial $6x^2 + 12x + 27$ in the class, can be factorize as $3(2x^2 + 4x + 9)$, we can say that 747 is composite. Moreover, $249 = 2x^2 + 4x + 9 = 24x + 9 = 3(8x + 3)$, in the class Ω_{249}; so, $747 = 3(2x^2 + 4x + 9) = 3(20x + 4x + 9) = 3(24x + 9) = 3[3(8x + 3)] = 3^2(8x + 3) = 3^2 \cdot 83$. Since 3 and 83 are prime numbers, we have found the representation of 747 as a product of primes without using division at all.

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[5] See my work: *How to Express Counting Numbers as a Product of Primes. Beginning Abstract Algebra at Elementary School.* *http://matematicasyfilosofiaenelaula.info/articulos.htm*

We understand by factoring the inverse process of multiplication in the Ω_a families. In other words, this is the process of recovering a and b, whenever we know that $c = a \cdot b$, with, $a > 1$ and $b > 1$.

Example

If we previously know that 1633 is the product of two factors a and b, then we may recover those factors, just factoring 1633 as a product of two factors. Indeed, inside the class Ω_{1633}, we can establish:

$1633 = x^3 + 6x^2 + 3x + 3 = 10x^2 + 6x^2 + 3x + 3 = 16x^2 + 3x + 3 = 15x^2 + 13x + 3 = 14x^2 + 23x + 3 = 14x^2 + 21x + 2x + 3 = 7x(2x + 3) + (2x + 3) = (7x + 1)(2x + 3)$.

We can choose $a = 7x + 1 = 71$ and $b = 2x + 3 = 23$. This shows that, $1633 = 71 \cdot 23$ and so 1633 is composite.

Definition above of composite numbers can be transformed by contraposition as "**if none of polynomials in the class Ω_a factorizes then** a **is not a composite number**", namely, a **is a prime number**. This last statement is a nice criterion to check primality.

Example

Let´s check 107 and 149 for primality using the above criterion.

$\Omega_{107} = \{x^2 + 7, 10x + 7, \ldots\}$

$\Omega_{149} = \{x^2 + 4x + 9, 14x + 9, \ldots\}$

None of polynomials inside the families Ω_{107} and Ω_{149} can be factored, then 107 and 149 are prime numbers. The reason for the first case is that the factors of x^2 are 1, 2, 5, 10, 100 and the factors of 7 are 1 and 7, do not have common factors. For the second polynomial applies similar considerations.

An example taken from a recent article [6] asks for factors of polynomial, $f(x) = x^8 + x^7 + x^5 + x^3 + x - 1$ in $\mathbf{Z}[x]$, the set of all polynomials with coefficients in $\mathbf{Z}$. We can associate this polynomial to the number 110.101.009, assuming that $x = 10$. The problem here would be factoring this number first, and then, try to discover through its factors, the polynomials associated to these numbers.

When I ask for the prime factors of this number to Wolfram app in my phone, I got the numbers: 73, 101, 109, and 137. These numbers can be associated to the polynomials $7x + 3$, $x^2 + 1$, $x^2 + 9$, $x^2 + 3x + 7$. However, the authors of the paper just found, $x^2 + 1$, $x^2 + x - 1$ and $x^4 + 1$, as irreducible factors of $f(x)$.

[6] Ayad, M. et al. *When Does a Given Polynomial with Integer Coefficients Divide Another*? **The American Mathematical Monthly.** Vol. 123, No. 4 April 2016, *Pag*. 380.

My explanation for this difference is that, first of all, this polynomial is not in $\mathbb{N}[x]$. Besides, $x^2 + x – 1$ and $x^2 + 9$ are quite different polynomials in $\mathbf{Z}[x]$ and we cannot apply the same rules as in $\mathbb{N}[x]$. Moreover, using the syntactic rule, $x^n = 10x^{n-1}$, the product $(7x + 3)(x^2 + 3x + 7) = x^4 + 1$, however, this equality is not quite true in $\mathbf{Z}[x]$.

The polynomial $x^4 + 1$ is irreducible in the ring $\mathbf{Z}[x]$ of polynomials with integer coefficients. However, the natural number 10001 associated to this polynomial, factors as $73 \cdot 137$, as you can see in Ω_{10001}.

$10001 = x^4 + 1 = 100x^2 + 1 = 99x^2 + 10x + 1 = \ldots = 91x^2 + 88x + 3 \cdot 7 = 7 \cdot 13x^2 + 13 \cdot 3x + 7 \cdot 7x + 3 \cdot 7 = 13x(7x + 3) + 7(7x + 3) = (7x + 3)(13x + 7) = 73 \cdot 137$

Consequently, 110.101.009 factors as $73 \cdot 101 \cdot 109 \cdot 137$, while, $x^8 + x^7 + x^5 + x^3 + x – 1 = (x^2 + 1)(x^2 + x – 1)(x^4 + 1)$ on $\mathbf{Z}[x]$.

As the example above shows, it is not always true that, $P(x) = R(x)S(x)$ in Ω_a, implies that $P(x)$ factors the same way in $\mathbf{Z}[x]$.

Example

Find the factors, if any, of $f(x) = x^4 + x^3 + 4x^2 + 4x + 9$. This polynomial is the standard polynomial of $a = 11449$. In Ω_a we can establish the following equalities.

$11449 = x^4 + x^3 + 4x^2 + 4x + 9 = 114x^2 + 4x + 9 = \ldots = 100x^2 + 140x + 49 = (10x)^2 + 2(10)(7)x + 7^2 = (10x + 7)^2 = (x^2 + 7)^2 = 107^2$.

The number $11449 = 107 \cdot 107$. However, $f(x)$ is irreducible in Z[x] because all zeros of $f(x)$ are complex, indeed, $x^2 + 7$ has not real roots. We can also realize that 107 is a prime number associated to an irreducible polynomial on N[x] as we showed above.

The following example suggests a way how to find the factors of some kind of polynomials $f(x)$, and so to solve the equation $f(x) = 0$.

Example

Let us factor the polynomial $f(x) = x^5 – 10x^4 + 35x^3 – 50x^2 + 24x$ inside Ω_a, extended to polynomials in Z[x], knowing that it is possible to do it.

First of all, we shall transform this polynomial into a polynomial where all coefficients be nonnegative integers, using the syntactic rule, $x^n = 10x^{n-1}$.

$x^5 – 10x^4 + 35x^3 – 50x^2 + 24x = 10x^4 – 10x^4 + 35x^3 – 50x^2 + 24x = 35x^3 – 50x^2 + 24x = 35x^3 – 5x^3 + 24x = 30x^3 + 2x^2 + 4x = 30240$.

$30x^3 + 2x^2 + 4x$ can be easily factored, as:

$30x^3 + 2x^2 + 4x = 2(15x^3 + x^2 + 2x) = 2(14x^3 + 10x^2 + 12x) = 2^2(7x^3 + 5x^2 + 6x) = 2^2(6x^3 + 14x^2 + 16x) = 2^3(3x^2 + 7x + 8)x = 2^3(2x^2 + 16x + 18)x = 2^4(x^2 + 8x + 9)x = 2^4(10x + 8x + 9)x = 2^4(18x + 9)x = 2^4 3^2(2x + 1)x = 2^4 3^2(21)x = 2^4 3^3(7)x = 2^4 3^3 \cdot 7x = 2^3 3^2(2\cdot 3)(2\cdot 5)(7)$.

Inside Ω_a we know that $x = 10 = 2\cdot 5$ and so: $1 = x - 9 = x - 3^2$; $2 = x - 8 = x - 2^3$; $3 = x - 7$; $4 = x - 6 = x - 2\cdot 3$, or:

$x = 2\cdot 5$; $x - 1 = 3^2$; $x - 2 = 2^3$; $x - 3 = 7$; $x - 4 = 2\cdot 3$.

Replacing above we get:

$x^5 - 10x^4 + 35x^3 - 50x^2 + 24x = (2^3)(3^2)(2\cdot 3)(2\cdot 5)(7) = (x - 2)(x - 1)(x - 4)\,x\,(x - 3)$.

The above process, show us how to factor some higher degree polynomials with the recourse of their numerical representation, **whenever the polynomial can be factored in Z[*x*]**.

In this particular case, we have also solved the equation: $x^5 - 10x^4 + 35x^3 - 50x^2 + 24x = 0$. The polynomials roots are: $x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 3$, and $x_5 = 4$.

Example. We can also solve elementary quadratic and linear equations without any knowledge of algebra. For instance, let us find numbers x such that $f(x) = x^2 - 7x + 10 = 0$.

The first step is to find the natural number associated to f through the definition of natural numbers extended to include negative coefficients. In Ω_a we can transform f as a polynomial with nonnegative coefficients and find their factors as in the above example.

$x^2 - 7x + 10 = 10x - 7x + 10 = 3x + 10 = 40 = 2^3\cdot 5 = (10-2)(10-5) = (x - 2)(x - 5)$.

We can verify easily that numbers 2 and 5 satisfy the equation.

Example

A cubic equation like $6x^3 + 25x^2 + 22x + 3 = 0$ has roots in **Q** and can be solved using the standard polynomial.

The first step in order to solve the equation is to find the standard polynomial associated to the above polynomial in the class Ω_a of equivalent polynomials module a.

$6x^3 + 25x^2 + 22x + 3 = 8x^3 + 5x^2 + 22x + 3 = 8x^3 + 5x^2 + 2x^2 + 2x + 3 = 8x^3 + 7x^2 + 2x + 3 = 8723$

The polynomial associated to the number 8723 help us to solve the equation.

The second step in the procedure is factoring the standard polynomial in Ω_a, as in the previous examples.

$8723 = 8x^3 + 7x^2 + 2x + 3 = 87x^2 + 2x + 3 = \ldots = 77x^2 + 102x + 3 = 77x^2 + 99x + 33 = 7\cdot 11x^2 + 9\cdot 11x + 3\cdot 11 = 11(7x^2 + 9x + 3) = 11(79x + 3) = 11(78x + 13) = 11(6\cdot 13x + 13) = 11\cdot 13(6x + 1) = 11\cdot 13\cdot 61 = (x + 1)(x + 3)(6x + 1)$.

Therefore

$6x^3 + 25x^2 + 22x + 3 = (x + 1)(x + 3)(6x + 1) = 0$. So, the polynomial vanishes when: $x = -1$, $x = -3$, $x = -1/6$.

Elementary algebra teach us that some equations do not have real solutions. That is the case of $x^2 + 4x + 9 = 0$, where the discriminant of the equation is negative. Here the standard polynomial associated to 149 is: $P(x) = x^2 + 4x + 9$. Inside Ω_{149} we find the following equivalent polynomials: $x^2 + 4x + 9$, $x^2 + 49$, $14x + 9$, none of them are expressible as a product of simpler polynomials. In this case we must conclude that $P(x)$ does not have rational roots. Note also that 149 is a prime number. In a general context, if the standard polynomial is associated to a prime number, the corresponding equation does not have any rational roots, whenever the polynomial in the equation be of grade greater than 1.

To finish this set of examples, let´s consider the case of the numbers triple (1,4374,4375) associated to the famous *abc conjecture* very publicized nowadays[7]. Those numbers have the rare property that the third one is greater than the product of the prime factors of $1\cdot4374\cdot4375 = 1\cdot2\cdot3^7\cdot7\cdot5^4$. The product of the prime factors of this number is: $2\cdot3\cdot5\cdot7 = 210$. Obviously, 210 is less than 4375.

We are going to find the prime factors of 4374 and 4375, through their respectively Ω families.

$4374 = 4x^3 + 3x^2 + 7x + 4 = 2x^3 + 22x^2 + 16x + 14 = 2(x^3 + 11x^2 + 8x + 7) = 2(21x^2 + 8x + 7) = 2(21x^2 + 6x + 27) = 2\cdot3(7x^2 + 2x + 9) = 2\cdot3(6x^2 + 12x + 9) = 2\cdot3^2(2x^2 + 4x + 3) = 2\cdot3^2(24x + 3) = 2\cdot3^3(8x + 1) = 2\cdot3^3(81) = 2\cdot3^3(3^4) = 2\cdot3^7$.

$4375 = 4x^3 + 3x^2 + 7x + 5 = 40x^2 + 35x + 25 = 5(8x^2 + 7x + 5) = 5(5x^2 + 35x + 25) = 5^2(x^2 + 7x + 5) = 5^2(15x + 2x + 5) = 5^3(3x + 5) = = 5^3(35) = 5^4\cdot7$.

That explains why $1\cdot4374\cdot4375 = 1\cdot2\cdot3^7\cdot5^4\cdot7$.

The above examples show some applications in connection with a new definition of natural numbers using polynomials in its most primitive form. The most important of all is the fact that we have made a support to justify the elementary procedure to find the prime factors of a natural composite number. I hope next time I would may show you more interesting applications in number theory and algebra.

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7 https://www.nature.com/articles/d41586-020-00998-2 . Issued on April 3, 2020.